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FIRST FUNDAMENTAL AXISYMMETRIC PROBLEM OF THERMOELASTICITY FOR A COMPRESSED SPHEROID WITH A CONCENTRIC SPHERICAL CAVITY

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An axisymmetric boundary-value problem of thermoelasticity for a compressed spheroid with a concentric spherical cavity is studied by the generalized Fourier method. The problem is reduced to an infinite system of linear algebraic equations with the Fredholm operator under the condition that the boundary surfaces are not crossed. Results of a numerical analysis of stresses in the case of load-free boundary surfaces in the presence of a temperature field caused by a constant temperature distribution on the boundary surfaces are presented.

Key words: *thermoelasticity, generalized Fourier method, multiply connected problems, Lamé equation.*

The fundamental boundary-value problems of the theory of thermoelasticity for a region bounded by spheroid and sphere surfaces were studied by the generalized Fourier method (GFM) in the absence of a thermal field [1]. This method is based on the use of theorems of addition of basis solutions of the Lamé equation for different canonical simply connected regions [2]. In the present work, we study the stress-strain state of a spheroid with a spherical cavity with allowance for thermoelastic strains caused by the action of an arbitrary axisymmetric steady temperature field. To solve this problem, the GFM for thermoelastic problems for multiply connected regions bounded by the surfaces of a sphere and a compressed spheroid was developed. The GFM for solving multiply connected thermoelastic problems was first developed in [3] for regions bounded by the surfaces of a hemisphere and a sphere. Results of the numerical analysis of stresses for various geometric parameters of the problem are reported in the present paper.

1. We introduce co-directed spherical (r, θ, φ) and compressed spheroidal (ξ, η, φ) coordinate systems fitted to the centers of the boundary surfaces. These coordinate systems are related through the formulas $r \sin\theta = c \cosh\xi \sin\eta$ and $r \cos\theta = c \sinh\xi \cos\eta$, where c is a spheroid parameter. The equations $r = R$ and $\xi = \xi_0$ define the boundary surfaces. The temperature T on the surface of the sphere and spheroid is set in the form of generalized Fourier series. The forces on the boundary surfaces are assumed to be known. The temperature distribution and the stress-strain state are determined by solving the following uncoupled steady problem of thermoelasticity:

$$\nabla^2 T = 0; \tag{1}$$

$$\nabla^2 \boldsymbol{U} + \frac{1}{1 - 2\sigma} \nabla(\nabla \boldsymbol{U}) = \frac{2(1 + \sigma)}{1 - 2\sigma} \, \alpha \nabla T; \tag{2}$$

$$T(R, \theta) = \sum_{n=0}^{\infty} a_n P_n(\cos\theta); \tag{3}$$

$$T(\xi_0, \eta) = \sum_{n=0}^{\infty} b_n P_n(\cos\eta); \tag{4}$$

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$$\boldsymbol{F}_R(R,\theta) = \sum_{j=1}^{2}\sum_{n=0}^{\infty} v_n^j P_n^{2-j}(\cos\theta)\,\boldsymbol{e}_j; \tag{5}$$

$$\boldsymbol{F}_\xi(\xi_0,\theta) = \sum_{j=1}^{2}\sum_{n=0}^{\infty} w_n^j P_n^{2-j}(\cos\eta)\,\boldsymbol{e}_j. \tag{6}$$

Here T is the temperature field, $\boldsymbol{U}$ is the vector of thermoelastic displacements, σ is Poisson's ratio, α is the coefficient of linear thermal expansion, $P_n(x)$ is the Legendre function, $\boldsymbol{F}_\xi$ and $\boldsymbol{F}_R$ are the vectors of forces on the surfaces of the spheroid and sphere, respectively, and $\{\boldsymbol{e}_j\}_{j=1}^2$ is the basis of the cylindrical coordinate system ($\boldsymbol{e}_\rho = \boldsymbol{e}_1$ and $\boldsymbol{e}_z = \boldsymbol{e}_2$).

The forces generated by the vector of thermoelastic displacements on an area with the normal vector $\boldsymbol{n}$ are determined as

$$\boldsymbol{F} = 2G\Big(\frac{\sigma}{1-2\sigma}\,\boldsymbol{n}\,\mathrm{div}\,\boldsymbol{U} + \frac{\partial \boldsymbol{U}}{\partial n} + \frac{1}{2}\,(\boldsymbol{n}\times\mathrm{rot}\,\boldsymbol{U}) - \frac{1+\sigma}{1-2\sigma}\,\alpha T\boldsymbol{n}\Big), \tag{7}$$

where G is the shear modulus.

The solution of the axisymmetric heat-conduction problem (1), (3), (4) is sought in the form of superposition of the spherical and spheroidal solutions

$$T = \sum_{n=0}^{\infty} C_n^1\Big(\frac{R}{r}\Big)^{n+1} P_n(\cos\theta) + \sum_{n=0}^{\infty} C_n^2\,\frac{P_n(iq)}{P_n(iq_0)}\,P_n(\cos\eta), \tag{8}$$

where C_n^1 and C_n^2 are unknown coefficients, $q = \sinh\,\xi$, and $q_0 = \sinh\,\xi_0$.

With the use of the GFM, the heat-conduction problem reduces to an infinite system of linear algebraic equations

$$\begin{gathered} C_n^1 + R^n \sum_{k=0}^{n} \frac{C_k^2}{P_k(iq_0)}\,A_n^k = a_n, \\ Q_n(iq_0)\sum_{k=n}^{\infty} C_k^1 R^{k+1} B_n^k + C_n^2 = b_n \qquad (n = 1, 2, \ldots), \end{gathered} \tag{9}$$

where A_n^k and B_n^k are coefficients in addition theorems reported in [4].

The solution of the inhomogeneous differential Lamé equation (2) is sought in the form of the sum of the general homogeneous solution $\boldsymbol{U}_0$ and the partial inhomogeneous solution. Since the right side of Eq. (2) depends on temperature, which, according to (8), can be represented as the sum of two components (spherical and spheroidal), we seek the partial solution of Eq. (2) in the form of the sum of two solutions corresponding to the spherical ($\boldsymbol{U}_1^T$) and spheroidal ($\boldsymbol{U}_2^T$) components of temperature:

$$\boldsymbol{U} = \boldsymbol{U}_0 + \boldsymbol{U}_1^T + \boldsymbol{U}_2^T.$$

The general solution of the homogeneous Lamé equation is sought in the form of superposition of external spherical and internal spheroidal basis solutions

$$\boldsymbol{U}_0 = \sum_{s=1}^{2}\Big(\sum_{n=0}^{\infty} a_n^s\,\frac{R^{n+1}}{n!}\boldsymbol{W}_{s,n}^{+} + \sum_{n=0}^{\infty} b_n^s\,\frac{1}{P_n(iq_0)}\,\boldsymbol{U}_{s,n}^{-}\Big),$$

where a_n^s and b_n^s are unknown coefficients; $\boldsymbol{W}_{s,n}^{\pm}$ and $\boldsymbol{U}_{s,n}^{\pm}$ are the axisymmetric variants of the general basis solutions of the Lamé equation for the sphere and compressed spheroid, which were constructed in [5] (the solutions for a compressed spheroid are obtained from the solutions for an extended spheroid by substituting $\xi + i\pi/2$ for ξ and $-ic$ for c); the superscripts plus and minus denote the external and internal solutions, respectively.

The partial solution of the inhomogeneous Lamé equation (2) corresponding to the spherical component of temperature is sought in the form

$$\boldsymbol{U}_1^T = \nabla r^2 \sum_{n=0}^{\infty} \alpha_n\,\frac{n!}{r^{n+1}}\,P_n(\cos\theta).$$

The unknown coefficients α_n are determined by substituting $\boldsymbol{U}_1^T$ into (2):

$$\alpha_n = \alpha \frac{1+\sigma}{2-2\sigma} \frac{C_n^1}{1-2n}.$$

The forces on the sphere surface generated by $\boldsymbol{U}_1^T$ and the spherical component of temperature (8) are found by formula (7) with the vector $\boldsymbol{e}_r = \boldsymbol{e}_1 \sin\theta + \boldsymbol{e}_2 \cos\theta$ taken as the normal vector. After transformations, we obtain

$$\boldsymbol{F}_1^T(R,\theta) = 2G\alpha \sum_{j=1}^{2}\sum_{n=0}^{\infty} \Big(\frac{1+\sigma}{2(1-\sigma)}\, \tau_{11,n}^j - \frac{1+\sigma}{1-2\sigma}\, T_{11,n}^j \Big) P_n^{2-j}(\cos\theta)\boldsymbol{e}_j,$$

$$\tau_{11,0}^j = C_0^1 \omega_{11,0}^{1j} + C_1^1 \omega_{11,1}^{1j}, \qquad \omega_{11,n}^j = \frac{C_{n-1}^1}{1-2n}\, \omega_{11,n-1}^{1j} - \frac{C_{n+1}^1}{1+2n}\, \omega_{11,n+1}^{2j} \qquad (n>1),$$

$$\omega_n^{1j} = \frac{\delta_{j1} + n\delta_{j2}}{2n+1} \Big(\frac{2\sigma}{1-2\sigma}\,(1-2n) + n(n-1) - \frac{n(n+1)}{2n+1} \Big),$$

$$\omega_n^{2j} = \frac{-\delta_{j1} + (n+1)\delta_{j2}}{2n+1} \Big(\frac{2\sigma}{1-2\sigma}\,(1-2n) + n(n-1) + \frac{n^2}{2n+1} \Big),$$

$$T_{11,0}^j = \delta_{j1} C_0^1 + \frac{C_1^1}{3}, \qquad T_{11,n}^j = \frac{1+n\delta_{j2}}{2n+3}\, C_{n+1}^1 + \frac{n\delta_{j2} - \delta_{j1}}{2n-1}\, C_{n-1}^1 \quad (n>1),$$

where δ_{kj} is the Kronecker delta.

To satisfy the boundary conditions on the spheroid surface (6), one has to find the expansions into the generalized Fourier series of forces generated by $\boldsymbol{U}_1^T$ and the spherical component of temperature (8) on the spheroid surface. For this purpose, $\boldsymbol{U}_1^T$ and the spherical component of temperature, written preliminary in the spheroidal coordinate system with the use of addition theorems described in [2, 5], are substituted into (7), where the external normal of the spheroid $\boldsymbol{e}_\xi = h(q\boldsymbol{e}_\theta \sin\eta + p\boldsymbol{e}_z \cos\eta)$ is used as the normal $[h = (q^2 + \cos^2\eta)^{-1/2}$ and $p = \cosh\xi]$. As a result, we obtain

$$\boldsymbol{F}_1^T(\xi_0,\eta) = 2\alpha G h \sum_{j=1}^{2}\sum_{n=0}^{\infty} \Big(\frac{1+\sigma}{2-2\sigma}\, \tau_{12,n}^j - \frac{1+\sigma}{1-2\sigma}\, T_{12,n}^j \Big) P_n^{2-j}(\cos\eta)\boldsymbol{e}_j,$$

$$\tau_{12,n}^j = \frac{2\sigma}{1-2\sigma}\,(q\delta_{j1} + p\delta_{j2})\omega_{12,n}^{1j} + cS_{1,n}^j + 2S_{2,n}^j,$$

$$\omega_{12,0}^{1j} = \delta_{j1} Q_0(iq)\Omega_{12,0}^1 + \frac{1}{3}\, Q_1(iq)\Omega_{12,1}^1, \qquad \Omega_{12,n}^1 = \sum_{k=0}^{n} C_k^1 R^{k+1} B_k^n,$$

$$\omega_{12,n}^{1j} = \frac{\delta_{j2} n + 1}{2n+3}\, Q_{n+1}(iq)\Omega_{12,n+1}^1 + \frac{\delta_{j2} n - \delta_{j1}}{2n-1}\, Q_{n-1}(iq)\Omega_{12,n-1}^1 \quad (n>1),$$

$$S_{1,n}^j = \omega_{12,n}^{2j} - 2p\Omega_{12,n}^2 \Big(qQ_n^{j-2}(iq) - p\,\frac{\partial Q_n^{j-2}(iq)}{\partial\xi} \Big), \qquad \Omega_{12,n}^2 = \sum_{k=1}^{n} \frac{C_{k-1}^1 R^k}{3-2k}\, \frac{(g^+)_k^n}{(k-1)!},$$

$$\beta_n^1 = \frac{n+1}{2n+1}, \qquad \beta_n^2 = \frac{n}{2n+1}, \qquad \gamma_{12,0}^j = \sum_{n=1}^{2} \beta_n^j \Omega_{12,n}^2\, \frac{\partial Q_n^{j-2}(iq)}{\partial\xi},$$

$$\gamma_{12,n}^j = \beta_{n+1}^j \Omega_{12,n+1}^2\, \frac{\partial Q_{n+1}^{j-2}(iq)}{\partial\xi} + \beta_{n-1}^{3-j}\Omega_{12,n-1}^2\, \frac{\partial Q_{n-1}^{j-2}(iq)}{\partial\xi} \qquad (n>1),$$

$$\omega_{12,0}^{2j} = \sum_{n=0}^{1} \gamma_{12,n}^j \beta_n^j, \qquad \omega_{12,n}^{2j} = \gamma_{12,n+1}^j \beta_{n+1}^j + \gamma_{12,n-1}^j \beta_{n-1}^{3-j},$$

$$\Omega_{12,n}^3 = \sum_{k=0}^{n} \frac{C_k^1}{1-2k}\, R^{k+1} B_k^n, \qquad \omega_{12,n}^{3j} = qQ_n^{j-1}(iq) + pQ_n^{2-j}(iq),$$

$$S^j_{2,0} = \delta_{j1}\omega^{3j}_{12,0} + \omega^{3j}_{12,1}/3,$$

$$S^j_{2,n} = \frac{\delta_{j2}n+1}{2n+3}\,\omega^{3j}_{12,n+1} + \frac{\delta_{j2}n-\delta_{j1}}{2n-1}\,\omega^{3j}_{12,n-1} \qquad (n>1),$$

$$T^j_{12,0} = (\delta_{j1}q+\delta_{j2}p)(\delta_{j1}\Omega^1_{12,0}Q_0(iq) + \Omega^1_{12,1}Q_1(iq)),$$

$$T^j_{12,n} = (\delta_{j1}q+\delta_{j2}p)\Big(\frac{\delta_{j1}+\delta_{j2}n}{2n+1}\,\Omega^1_{12,n+1}Q_{n+1}(iq) + \frac{\delta_{j2}(n+1)-\delta_{j1}}{2n-1}\,\Omega^1_{12,n+1}Q_{n+1}(iq)\Big) \quad (n>1).$$

Here $(g^{\pm})^k_n$ are the coefficients in vector theorems of addition of the general basis solutions of the Lamé equation for the sphere and compressed spheroid, which were obtained in [2, 5].

We seek the partial solution of the inhomogeneous Lamé equation (2) corresponding to the spheroidal component of temperature (8) in the form

$$\boldsymbol{U}^T_2 = \sum_{n=0}^{\infty}\beta_n z\boldsymbol{U}^-_{1,n}.$$

The coefficients β_n are determined by substituting $\boldsymbol{U}^T_2$ into the Lamé equation (2):

$$\beta_n = \alpha\,\frac{2-2\sigma}{4\sigma-3}\,\frac{C^2_n}{P_n(iq_0)}.$$

The generated forces are found by formula (7) with $\boldsymbol{e}_\xi$ used as the normal. As a result, we obtain

$$\boldsymbol{F}^T_2(\xi_0,\eta) = 2G\alpha h\sum_{j=1}^{2}\sum_{n=0}^{\infty}\Big(\frac{2-2\sigma}{4\sigma-3}\,\tau^j_{22,n} - \frac{1+\sigma}{1-2\sigma}\,T^j_{22,n}\Big)P^{2-j}_n(\cos\eta)\boldsymbol{e}_j,$$

$$\tau^j_{22,0} = \frac{C^2_0\omega^{1j}_{22,0}}{P_0(iq_0)} + \frac{C^2_1\omega^{1j}_{22,1}}{P_1(iq_0)}, \qquad \tau^j_{22,n} = \frac{C^2_{n-1}\omega^{1j}_{22,n-1}}{P_{n-1}(iq_0)} + \frac{C^2_{n+1}\omega^{2j}_{22,n+1}}{P_{n+1}(iq_0)},$$

$$\omega^{j1}_{22,n} = \frac{1}{2n+1}\Big(\chi^1_n(n+\delta_{j1}) - (\delta_{j1}-\delta_{j2})\,\frac{\sigma}{1-2\sigma}\,qP_n(iq)\Big),$$

$$\omega^{j2}_{22,n} = \frac{n+\delta_{j2}}{2n+1}\Big(\chi^2_n + (\delta_{j1}-\delta_{j2})(n+\delta_{j1})\,\frac{qP^{-1}_n(iq)}{2}\Big),$$

$$\chi^1_n = \Big(\frac{q^2}{p}\,(n+1) - \frac{p}{2}\Big)P^{-1}_n(iq) + i\,\frac{q}{p}\,(n+1)P^{-1}_{n+1}(iq),$$

$$\chi^2_n = -qP^1_n(iq) - \frac{1-\sigma}{1-2\sigma}\,pP_n(iq), \qquad T^j_{22,0} = (\delta_{j1}q+\delta_{j2}p)\Big(\delta_{j1}C^2_0 + \frac{1}{3}\,C^2_1\Big),$$

$$T^j_{22,n} = (\delta_{j1}q+\delta_{j2}p)\Big(\frac{\delta_{j1}+\delta_{j2}n}{2n+3}\,C^2_{n+1} + \frac{\delta_{j2}n-\delta_{j1}}{2n-1}\,C^2_{n-1}\Big) \qquad (n>1).$$

Using the addition theorems, we can write the forces on the sphere surface, which correspond to $\boldsymbol{U}^T_2$ and the spheroidal component of temperature, can be written as

$$\boldsymbol{F}^T_2(R,\theta) = 2G\alpha\sum_{j=1}^{2}\sum_{n=0}^{\infty}\Big(\frac{2-2\sigma}{4\sigma-3}\,\tau^j_{21,n} - \frac{1+\sigma}{1-2\sigma}\,T^j_{21,n}\Big)P^{2-j}_n(\cos\theta)\boldsymbol{e}_j,$$

$$\omega^j_{21,0} = \sum_{k=0}^{1}R^k\Omega^1_{21,k}\omega^{2j}_{21,k},$$

$$\tau^j_{21,n} = \frac{R^{n-1}}{(n-1)!}\,\Omega^1_{21,n-1}\omega^{2j}_{21,n-1} + \frac{R^{n+1}}{(n+1)!}\,\Omega^1_{21,n+1}\omega^{1j}_{21,n+1},$$

$$\omega^{j1}_{21,n} = \frac{\sigma}{1-2\sigma}\,\frac{\delta_{j2}-\delta_{j1}}{2n+1} + \Big(\frac{n}{2n+1} + \frac{1}{2n+2}\Big)\frac{n+\delta_{j2}}{2n+1},$$

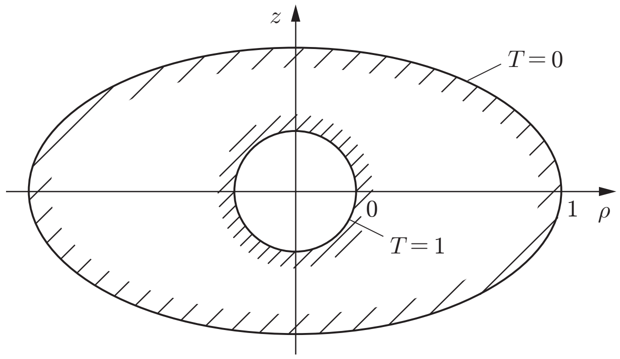

Fig. 1

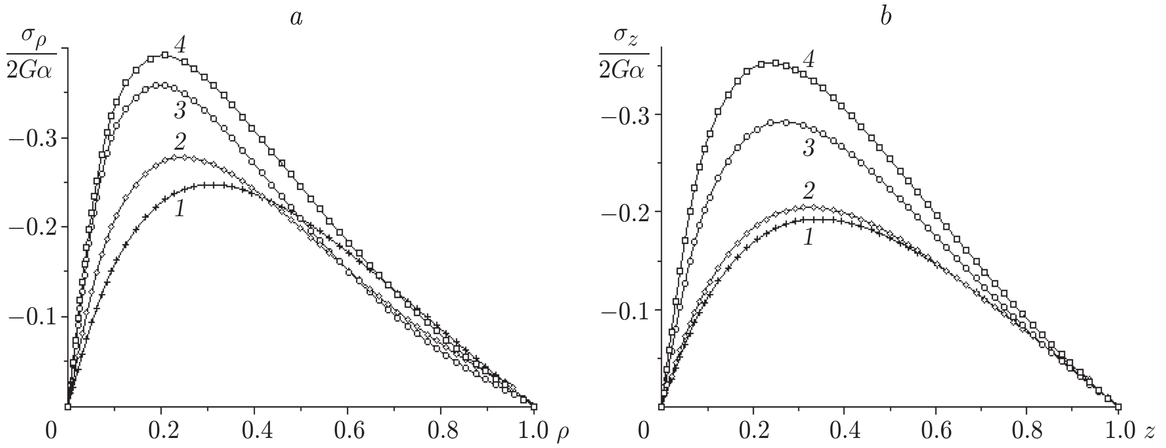

Fig. 2

$$\omega^{j2}_{21,n} = \frac{n(n+1)}{2n+1}\,\frac{\delta_{j1}-\delta_{j2}}{2n+2} + \Big(\frac{1-\sigma}{1-2\sigma} + n\Big)\frac{n+\delta_{j1}}{2n+1},$$

$$\Omega^{1}_{21,n} = -\sum_{k=n}^{\infty} \frac{C^2_k}{P_n(iq_0)}\,(g^-)^n_k,$$

$$T^{j}_{21,0} = \delta_{j1}\Omega^T_{21,0} + \frac{\Omega^T_{21,1}}{3}, \qquad \Omega^T_{21,n} = \sum_{k=n}^{\infty} \frac{C^2_k}{P_n(iq_0)}\,A^n_k,$$

$$T^{j}_{21,n} = \frac{1+n\delta_{j2}}{2n+3}\,R^{n+1}\Omega^T_{21,n+1} + \frac{n\delta_{j2}-\delta_{j1}}{2n-1}\,R^{n-1}\Omega^T_{21,n-1} \qquad (n>1).$$

Satisfying the conditions on the boundary surfaces (5) and (6), we obtain an infinite system of linear algebraic equations with respect to the unknown coefficients a^s_n and b^s_n:

$$\sum_{i=1}^{2}\Big(s^i_{n,j}a^i_n + \sum_{k=0}^{n} t^{i,j}_{n,k}b^i_k\Big) = A^1_{n,j}, \qquad \sum_{i=1}^{2}\Big(\sum_{k=n}^{\infty} s^{i+2,n}_{n,j}a^i_n + t^{i+2}_{n,j}b^i_n\Big) = A^2_{n,j} \tag{10}$$

$$(j=1,2,\ n=1,2,\ldots).$$

The coefficients of system (10) are found from the coefficients of a similar system for an extended spheroid, which are given in [1], by substituting $a=0$ and replacing ξ by $\xi+i\pi/2$ and c by $-ic$. As in [1], it can be shown with the help of the estimates of [6] that the operators of systems (9), (10) are the Fredholm operators if the boundary surfaces are not crossed, which allows one to solve the system by the reduction method.

In system (10), the right sides have the form

$$A_{n,j}^1 = v_n^j - \alpha \frac{1+\sigma}{2(1-\sigma)} \tau_{11,n}^j - \alpha \frac{2-2\sigma}{4\sigma-3} \tau_{21,n}^j + \alpha \frac{1+\sigma}{1-2\sigma} (T_{11,n}^j + T_{21,n}^j),$$

$$A_{n,j}^2 = w_n^j - \alpha \frac{1+\sigma}{2(1-\sigma)} \tau_{12,n}^j - \alpha \frac{2-2\sigma}{4\sigma-3} \tau_{22,n}^j + \alpha \frac{1+\sigma}{1-2\sigma} (T_{22,n}^j + T_{12,n}^j).$$

2. The numerical analysis of the problem was performed for the following parameters: $R = 1$ and $\sigma = 0.25$. The boundaries are load-free; the temperature of the inner surface ($r = R$) equals unity, and the temperature of the outer surface $\xi = \xi_0$) equals zero (Fig. 1).

Figure 2a shows the distributions of thermoelastic stresses on areas normal to $\boldsymbol{e}_r$ in the focal plane of the spheroid. Figure 2b shows the distributions of thermoelastic stresses on areas located on the z axis and normal to $\boldsymbol{e}_z$. Curves 1–4 refer to the lengths of the major and minor axes of the spheroid (2.0 and 1.5), (2.5 and 1.5), (10/3 and 2.0), and (10/3 and 2.5), which are obtained for the following values of ξ_0 and c: (1.32 and 0.973), (2.0 and 0.693), (2.667 and 0.693), and (2.205 and 0.973).

The results of studying the stress-strain state show that the character of stress distribution depends on the distance between the boundary surfaces in a chosen direction, and the values of stress are minimum both in the case where the spheroid size is close to the sphere size (curves 1 and 2) and in the case where the spheroid size is an order of magnitude greater than the sphere size. The latter can be explained by the smaller influence of the outer surface on the stress-strain state in the vicinity of the sphere: the spheroid does not affect the sphere.

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